Helioseismology

In the 1960's, it was found that turbulence in the Sun's convective zone created waves below the surface. (Essentially, the convective motions were ringing a bell). Certain frequencies become amplified by constructive interference, leading to resonances, while others are destroyed by destructive interference. The interpretation is that standing waves are trapped in cavities, with one end of the cavity being close to the stellar photosphere, and the other at a depth depending on the frequency. These waves manifest themselves as ~ 1 m/s pulsations motions across the surface of the Sun. The typical period of these pulsations is ~ 5 min, though the power spectrum is complex, due to the beating of *many, many* periods.

Consider two waves leaving A and arriving in B in phase. An oscillation will be seen provided the paths differ by an exact number of wavelengths. In that case, the waves will reinforce each other; otherwise, they'll interfere and have negligible amplitude.

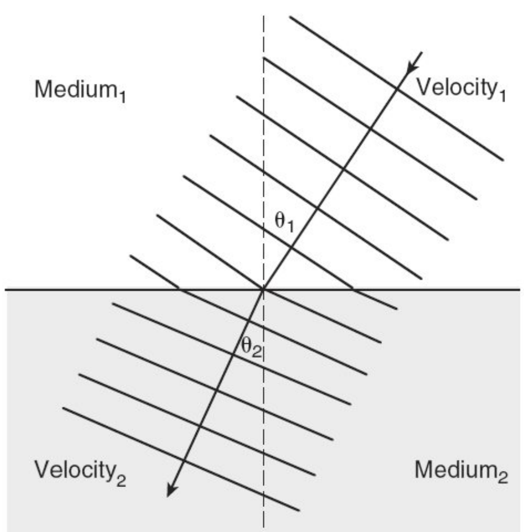
There are three types of oscillations that may occur in a star.

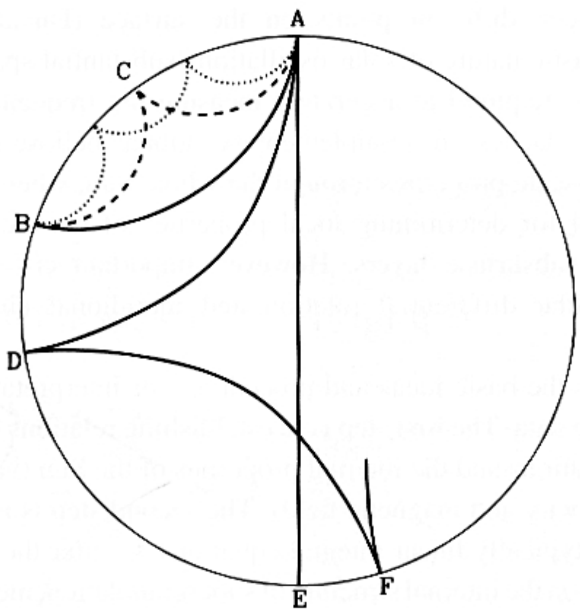
p-modes

p-mode oscillations are simply sound waves moving through gas, i.e., their restoring force is the local pressure. The speed of sound plus the ideal gas law implies

$$c_s = \left(\frac{\gamma P}{\rho}\right)^{1/2} = \left(\frac{\gamma k T}{\mu m_H}\right)^{1/2} \propto \sqrt{\frac{T}{\mu}} \qquad (32.01)$$

So the deeper into the star the wave goes, the larger the sound speed. In other words, for a wave traveling at an angle, the deeper wave front will always be traveling faster than the trailing wave front. This leads to refraction.





For a p-mode to create a standing wave, it must reflect off the stellar surface. If the pressure scale height, λ_P, is larger than the wavelength of a mode, the wave has no trouble propagating and is transmitted. However, if the λ_P is shorter than the wave, the gas readjusts on a timescale that is shorter than the wave's period, and the wave is reflected. The upshot is that there is a maximum

frequency to observable p-modes:

$$\nu_{\text{cut}} = \frac{c_s}{\lambda_P} = c_s \bigg/ \left(\frac{dr}{d \ln P} \right) \tag{32.02}$$

Since the star is in approximate hydrostatic equilibrium, and we are concerned with the stellar surface

$$\frac{dP}{dr} = \frac{G\mathcal{M}}{R^2} \rho \tag{32.03}$$

which implies

$$\begin{aligned}
\nu_{\text{cut}} &= c_s \frac{G\mathcal{M}_T}{R^2} \frac{\rho}{P} \\
&= \left(\frac{\gamma P}{\rho} \right)^{1/2} \frac{G\mathcal{M}_T}{R^2} \frac{\rho}{P} \\
&= \left(\frac{\gamma \mu m_H}{k} \right)^{1/2} \frac{G\mathcal{M}_T}{R^2} \left(\frac{k T_{\text{eff}}}{\mu m_H} \right)^{-1} \propto g T_{\text{eff}}^{-1/2}
\end{aligned} \tag{32.04}$$

p-modes are likely excited through stochastic acoustic noise from fluctuating turbulent pressure and gas pressure in the Sun's convective layer. Radiative losses, viscosity, and non-linear interactions between modes all work to dampen the oscillations. For low ℓ values, a particular p-mode will decay exponentially on a timescale of days, suggesting

1) whatever excites the p-modes happens on a timescale of a few days.

2) destructive interference has plenty of time to destroy all the non-resonant frequencies, changing random convective noise into a very rich power spectrum.

In general, low frequencies waves are damped out more slowly than high frequency waves. The lowest frequency modes may take months to decay.

g-modes

g-modes oscillations are driven by the negative buoyancy of vertically displayed material. Recall that the condition for convective stability is

$$\left(\frac{\delta}{T}\frac{dT}{dr}\right)_i < \left(\frac{\delta}{T}\frac{dT}{dr} - \frac{\phi}{\mu}\frac{d\mu}{dr}\right)_s \implies \nabla_{\rm rad} < \nabla_{\rm ad} \qquad (3.2.5)$$

This can be rewritten as

$$\Delta\rho = \frac{\rho\delta}{\lambda_P}\left[\nabla_{\rm ad} - \nabla_{\rm rad} + \frac{\phi}{\delta}\frac{d\ln\mu}{d\ln P}\right] \qquad (32.05)$$

The buoyancy acceleration associated with the density perturbation is therefore

$$\frac{\partial^2\Delta r}{\partial t^2} = -\frac{g\delta}{\lambda_P}\left[\nabla_{\rm ad} - \nabla_{\rm rad} + \frac{\phi}{\delta}\frac{d\ln\mu}{d\ln P}\right] \qquad (32.06)$$

If we neglect the term associated with the chemical gradient, then an element which is displaced by a distance Δr in a region of stability, will move with $\Delta r = \Delta r_0 e^{iNt}$ with a frequency

$$\begin{aligned} N^2 &= -\frac{g\delta}{\lambda_P}\left[\nabla_{\rm ad} - \nabla_{\rm rad} + \frac{\phi}{\delta}\frac{d\ln\mu}{d\ln P}\right] \\ &= g\left[\frac{1}{\gamma P}\frac{dP}{dr} - \frac{1}{\rho}\frac{d\rho}{dr}\right] \qquad (32.07) \end{aligned}$$

This is the Brunt-Väisälä frequency, or buoyancy frequency. The implication is that in regions of stability, material can oscillate and propagate g-modes, but in convective regions, displacements grow exponentially and no oscillations occur.

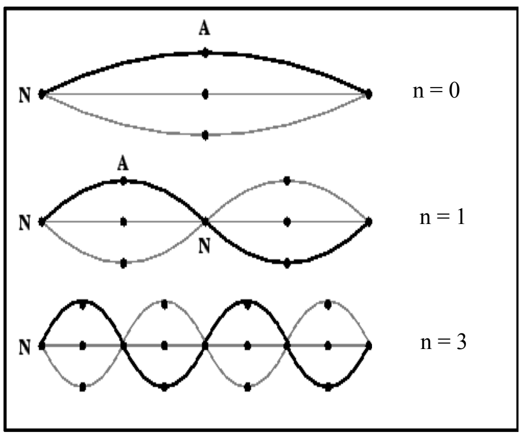
In the Sun, g-modes can only occur in the central (radiative) regions. They are most likely excited by nuclear burning instabilities, and have periods greater than 50 min.

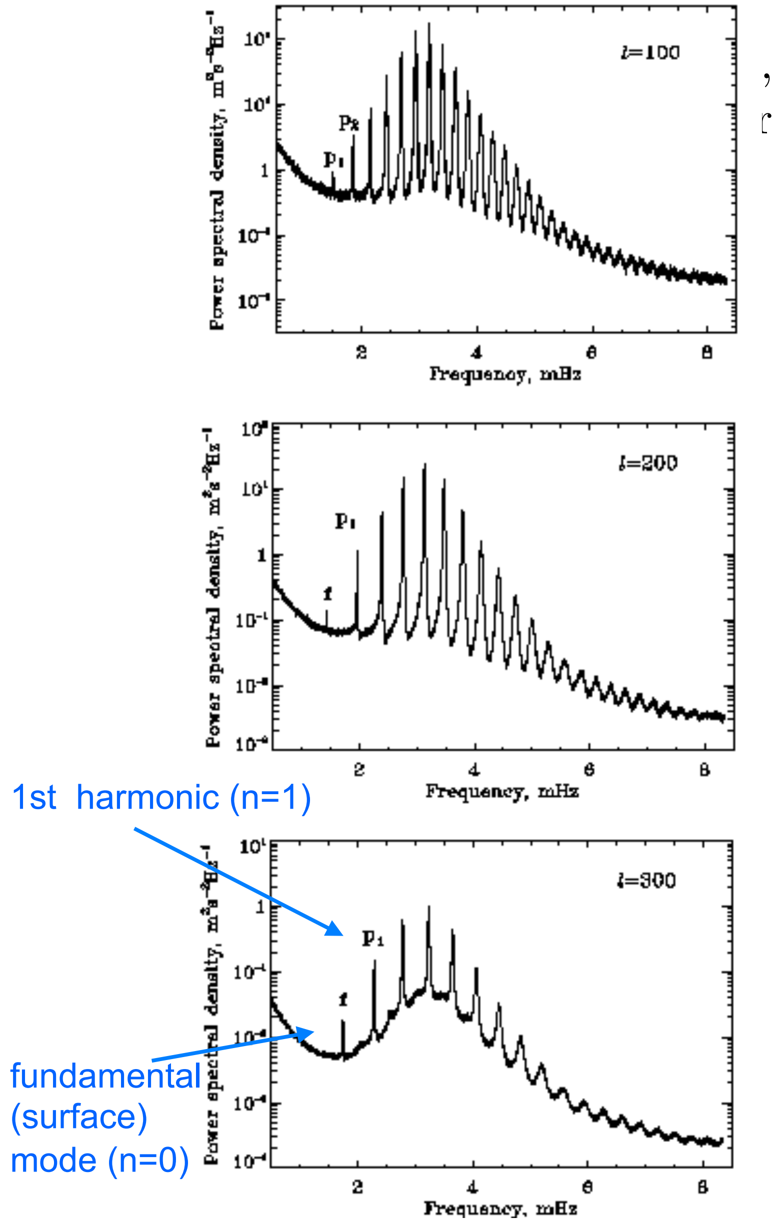
f-modes

f-modes are low-frequency surface waves propagating in their fundamental ($n = 0$) mode. As with stars in the instability strip, simple gravity is the restoring force. They are similar to deep sea waves, as the medium acts as an incompressible fluid. Since these waves never penetrate more than R/ℓ into a star (where R is the stellar radius), they do not provide much information about the stellar interior.

Pulsation Modes

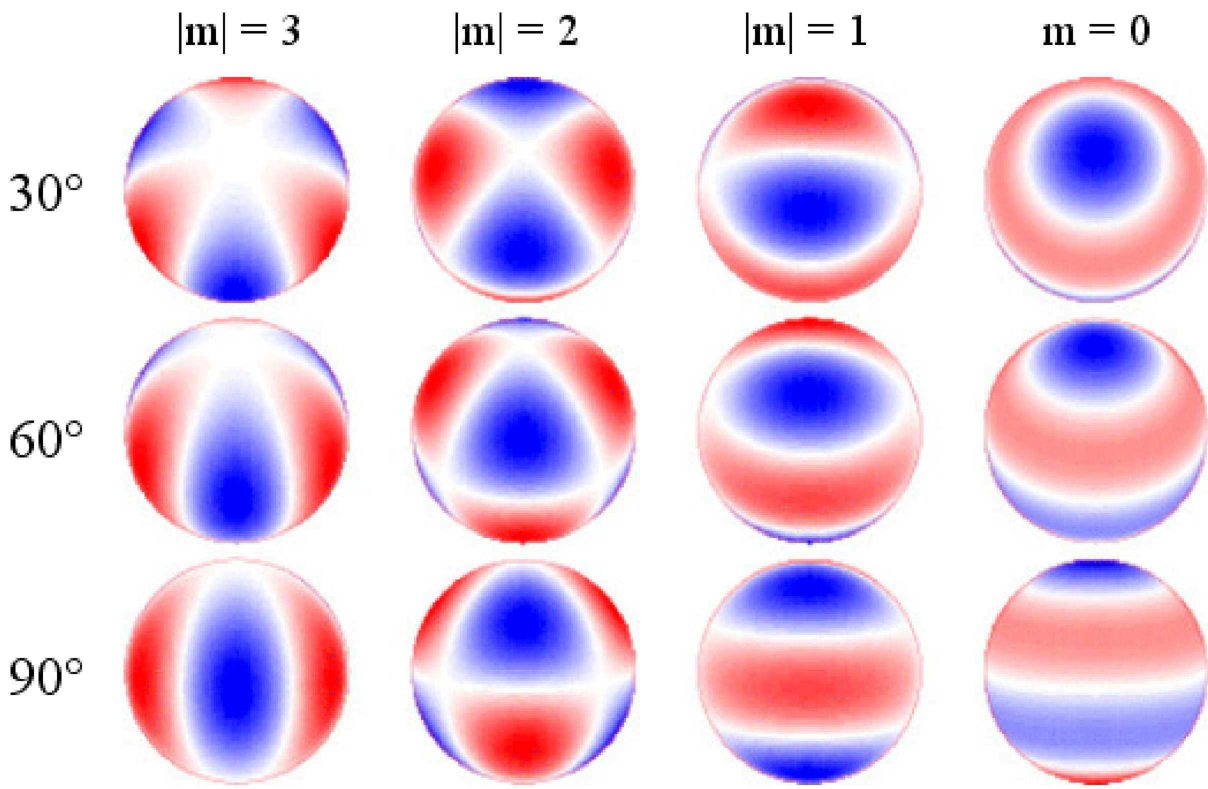
The pulsation modes of stars can be described by spherical harmonics and three integer numbers:



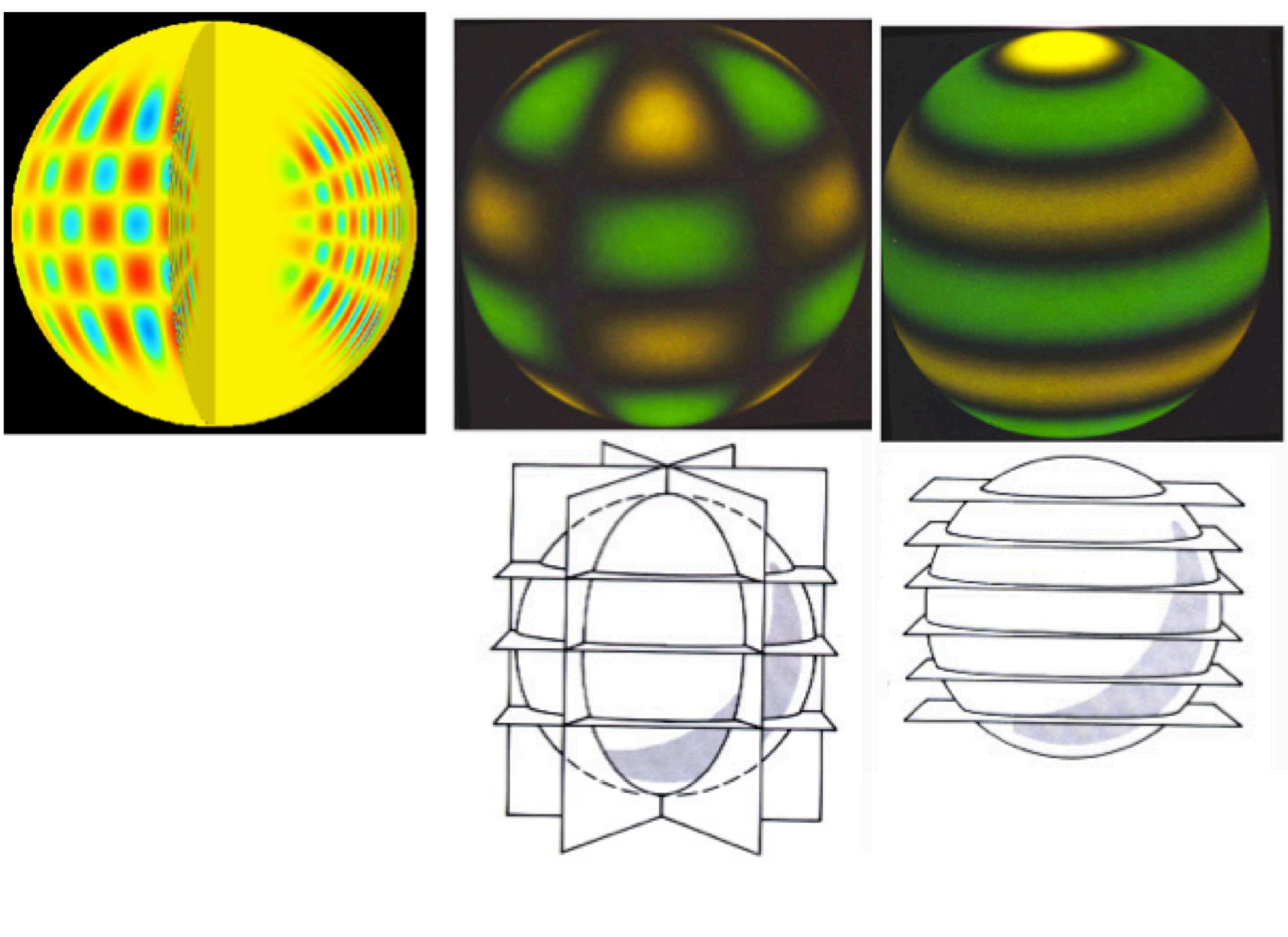


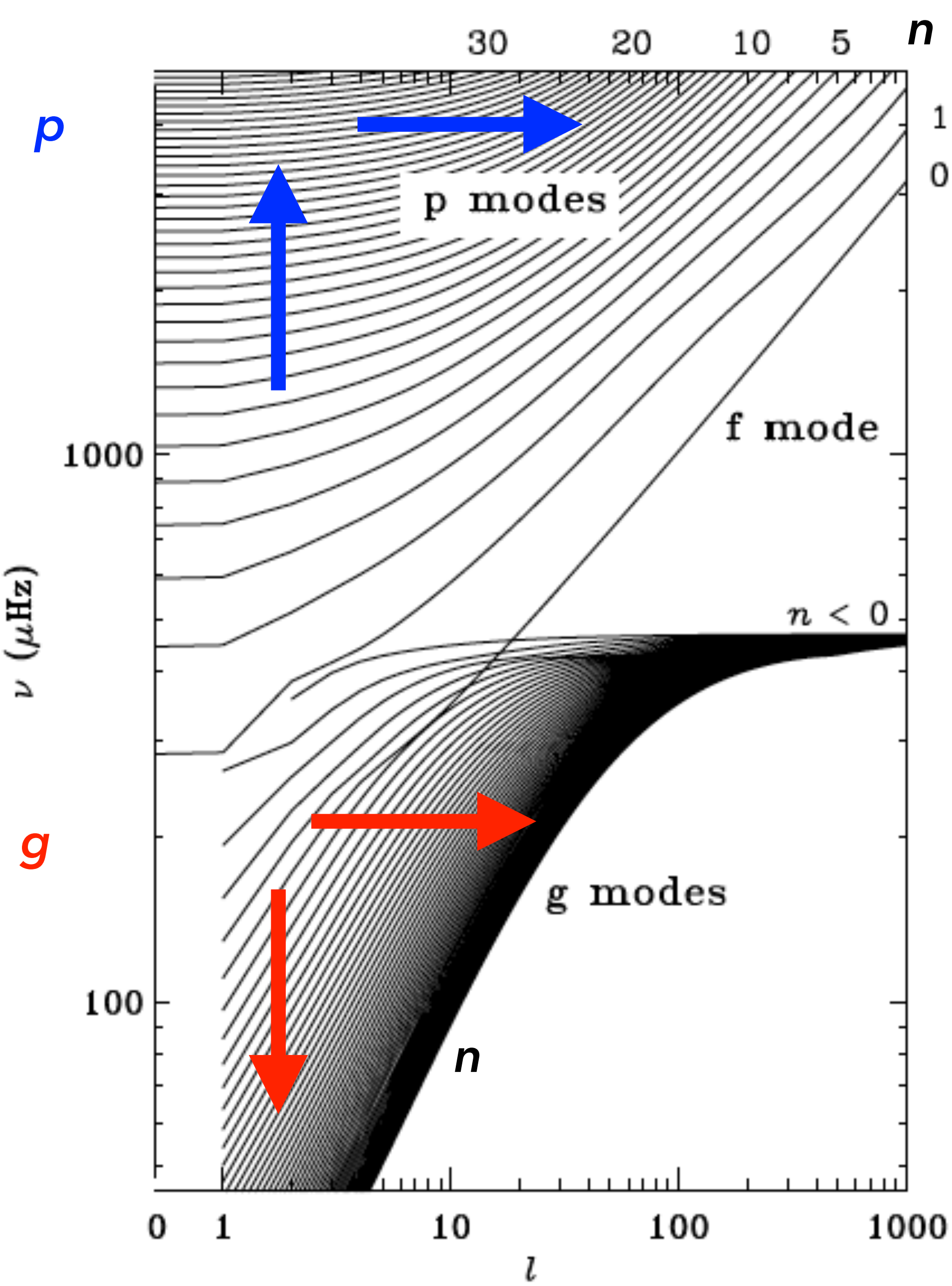
• The harmonic degree, ℓ, is the number of nodes lines on the surface. Alternatively, one can think of ℓ as the total number of planes slicing through the Sun. A purely radial pulsation has $\ell = 0$.

• the azimuthal order, m, gives how many of the surface node lines cross the equator. In effect, it describes how many planes slice through the Sun longitudinally. m ranges from $-\ell$ to $+\ell$.



Examples of $l = 3$ models seen from different inclinations: 30° (top row), 60° (middle row) and 90° (bottom row). Blue represents areas of the surface moving inward; red shows regions moving outward, and white displays the nodal surface lines. From left to right, the columns display $|m| = 3, 2, 1$, and 0.

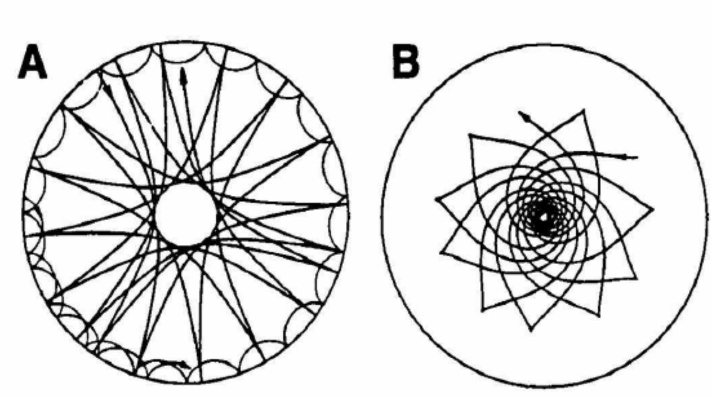


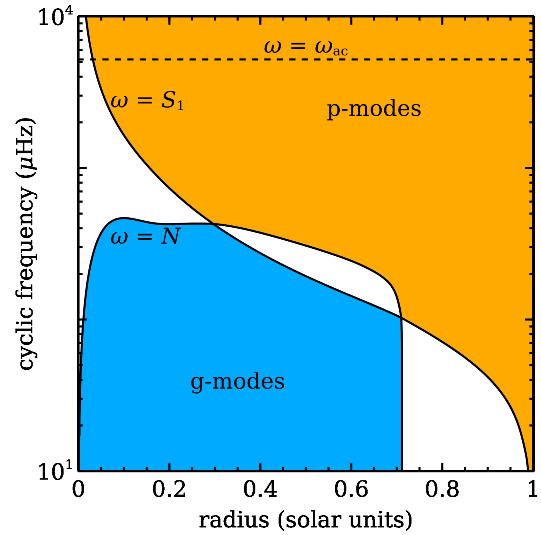


Note: being acoustic waves, p-mode frequencies increase with an increase of the number of radial nodes (n) and degree ℓ. But because buoyancy drives g-modes, their frequences decrease with larger n (but increase with ℓ).

The Sun

The Sun has a convective envelope and a radiative core. As a result, the Sun's g-modes are confined to the interior and are not directly observable. But $\sim 10^7$ separate p-modes and f-modes have been observed on the Sun's surface.



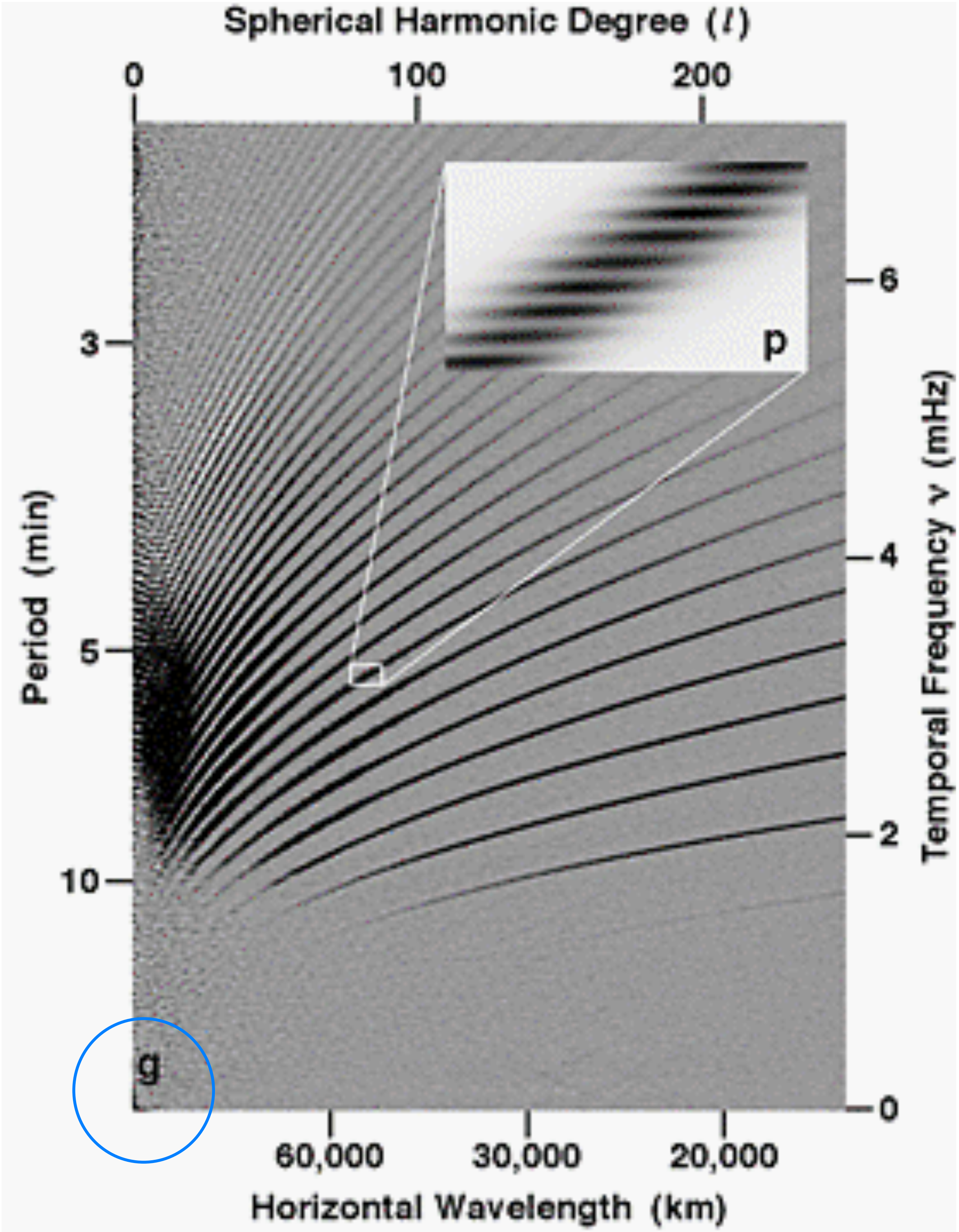


To measure p-modes in the Sun, one needs a long time series of observations (at least a month, ideally over several years) and obtain Doppler velocities (to better than ~ 0.1 m/s) over the Sun's surface. (Because we are measuring radial oscillations, the data are much more easily obtained for the center of the Sun's disk that it's edge.) Typically, measurements are made once a minute.

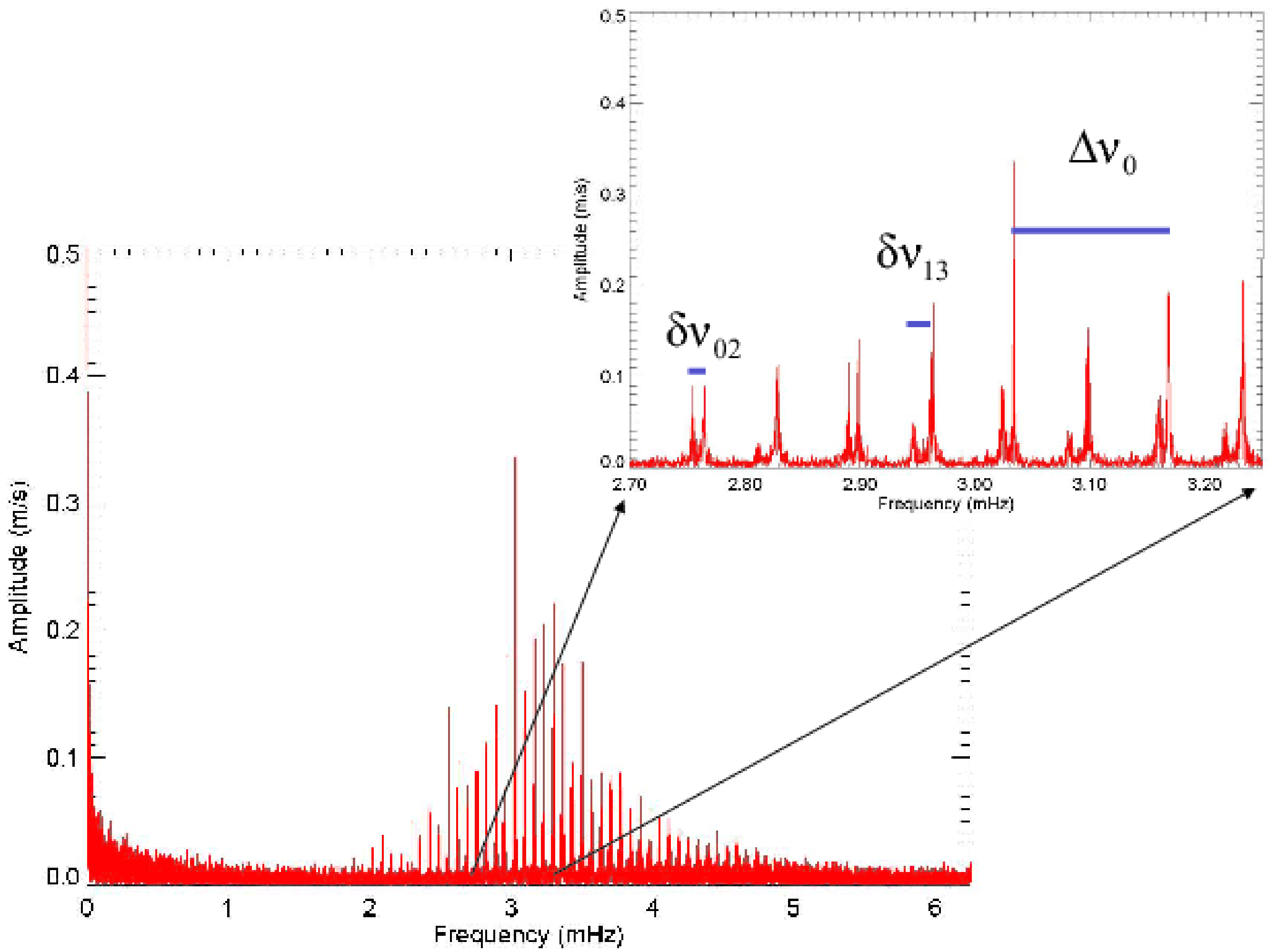
Velocity data are decomposed into spatial spherical harmonics,

$$a' = \sum_{\ell,m} = a_{\ell,m}(t) Y_\ell^m(\theta, \phi) \tag{32.08}$$

where θ is the co-latitude and ϕ is the longitude. The $a_{\ell,m}$ amplitudes contain all the information on the (ℓ, m) modes; a Fourier transform of $a_{\ell,m}(t)$ then gives the frequencies of oscillations of the modes, $a_{\ell,m}(\nu)$. The final result is a power spectrum of three variables: ν, ℓ, and m.



At a given ℓ, each peak (ridge) represents a mode with different n (i.e., nodes in the radial direction): the higher the frequency, the higher the value of n, and the higher the depth of penetration.

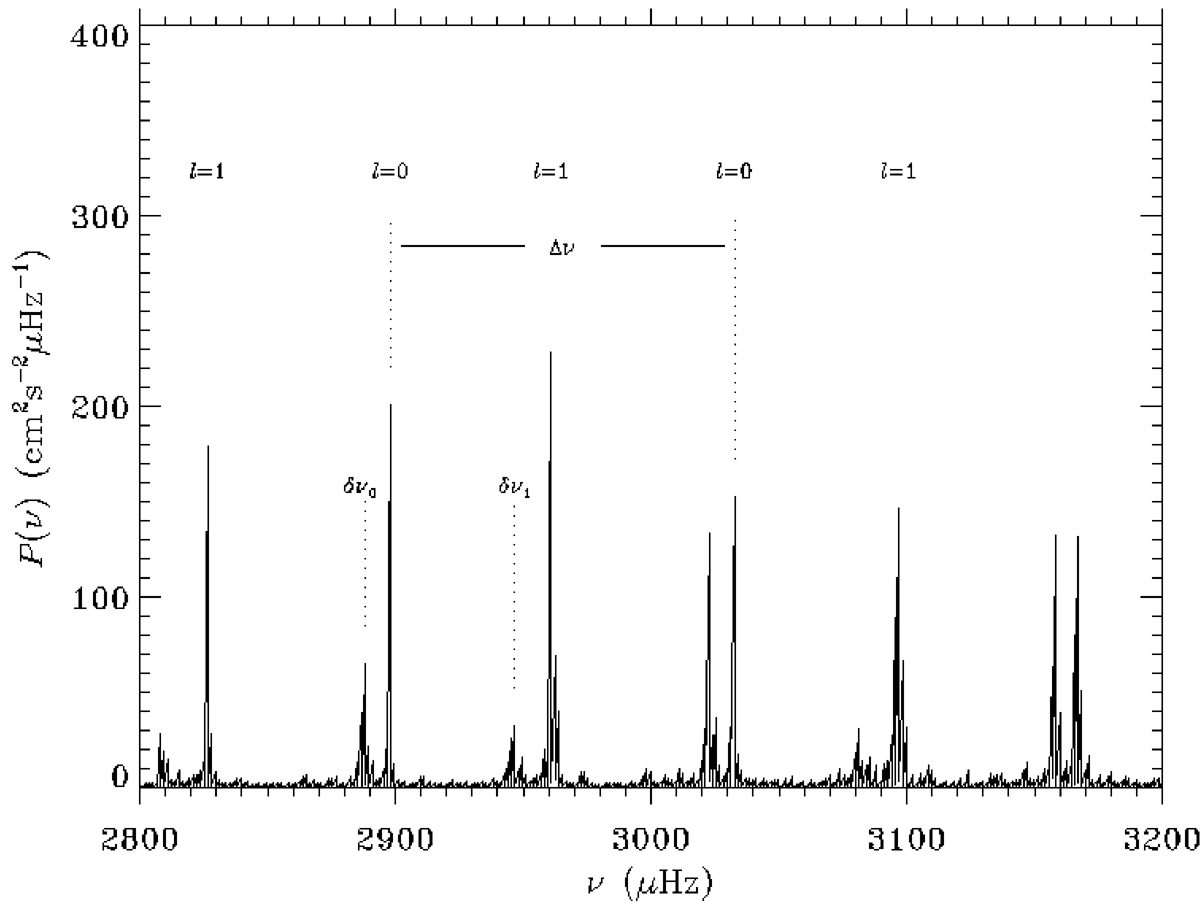


The pattern shows a relation between the frequency and the mode

$$\nu_{n,\ell} = \Delta\nu \left(n + \frac{\ell}{2} + \alpha + \frac{1}{4} \right) + \epsilon_{n,\ell} \tag{32.09}$$

where α is a function of the frequency determined by the properties of the surface layers, and $\epsilon_{n,\ell}$ is a small correction which depends on the conditions in the stellar core. $\Delta\nu$ is the inverse of the sound travel time across the stellar diameter:

$$\Delta\nu = 2 \left(\int_0^R \frac{dr}{c_s} \right)^{-1} \tag{32.10}$$



Note that from the virial theorum and the ideal gas law

$$c_s \propto \left(\frac{P}{\rho}\right)^{1/2} \propto T^{1/2} \propto \left(\frac{G\mathcal{M}}{R}\right)^{1/2} \tag{32.11}$$

so

$$\Delta\nu \propto \left(\frac{G\mathcal{M}}{R^3}\right)^{1/2} \propto \langle\rho\rangle^{1/2} \tag{32.12}$$

Because the propagation of p-modes depends on the sound speed (which, through the equation of state, depends on $\rho(r)$, $\mu(r)$, and $T(r)$), and because each mode penetrates to different layer of the Sun, the 2-D power spectrum can be used to obtain measure the internal structure and composition of the Sun. Moreover,

• For a static Sun, the modes shouldn't depend on the direction of wave propatation. However, rotation breaks this degeneracy, as

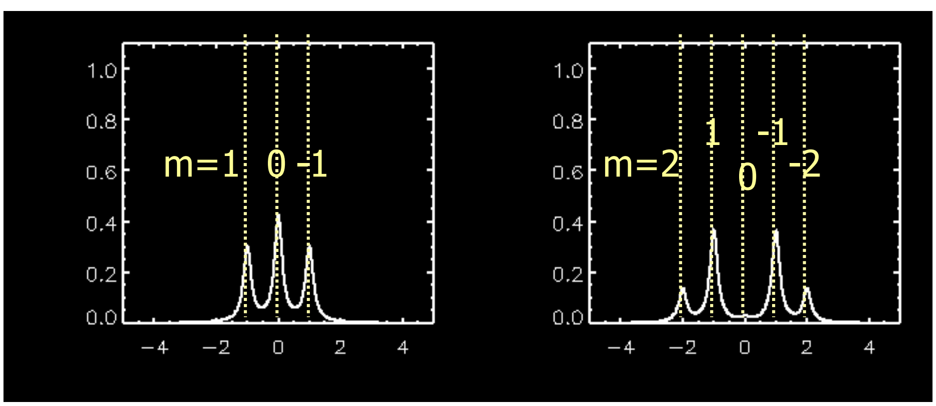
sounds will propagate faster in the direction of rotation than against the rotation. As a result, modes of the same (n,ℓ) but different m will be split into $2\ell+1$ frequency components. Specifically, a wave moving in the direction of rotation will have its frequency Doppler shifted by

$$\frac{\Delta\nu_+}{\nu} = \frac{v_+}{v_p} \tag{32.13}$$

where v_+ is the rotation velocity and v_p is the wave's phase velocity. Similarly, a wave moving against the rotation will be shifted by

$$\frac{\Delta\nu_-}{\nu} = \frac{v_-}{v_p} \tag{32.14}$$

Hence the two waves will be split in frequency by $2\Delta\nu$. The result is that the entire standing wave pattern drifts slowly across the Sun in the direction opposite the solar rotation. By examining how this shift changes with (n,ℓ), one can measure the Sun's rotation over 90% of its mass.



• For most applications, m splitting is measured, then removed, and the power spectra are averaged over the m states to produce the $\ell-\nu$ diagram.

• The sound speed can be affected by localized phenomenon, such as the sunspots. (Their lower temperatures lower the sound speed.)

Thus, one can "observe" activity on the far side of the Sun via the p- and f-modes, measure magnetic field strengths, etc.

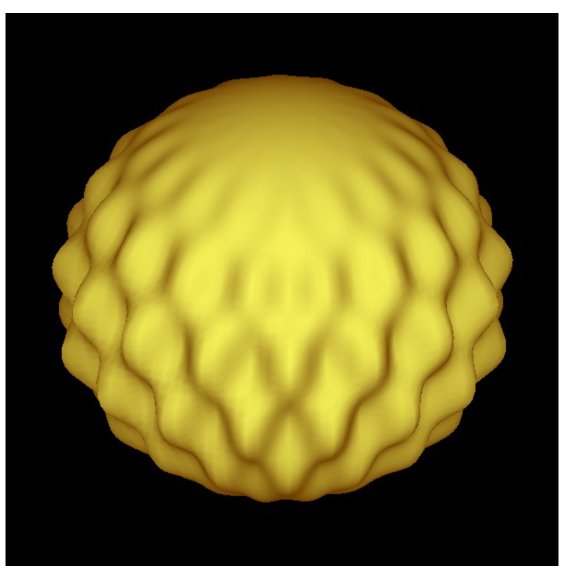
• Because the sound speed depends on the mean molecular weight, helioseismology can be used to determine the Sun's age: the larger the age, the greater the helium abundance, the larger the mean molecular weight, and the slower the sound speed.

• Another way of looking at pulsations is through the use of two critical frequencies, the Lamb frequency and the Brunt-Väisälä frequency. They can be written as

$$L_\ell = \frac{\ell(\ell+1)c_s^2}{r^2} \quad \text{and} \quad N = g\left(\frac{1}{P\gamma}\frac{dP}{dr} - \frac{1}{\rho}\frac{d\rho}{dr}\right) \tag{32.15}$$

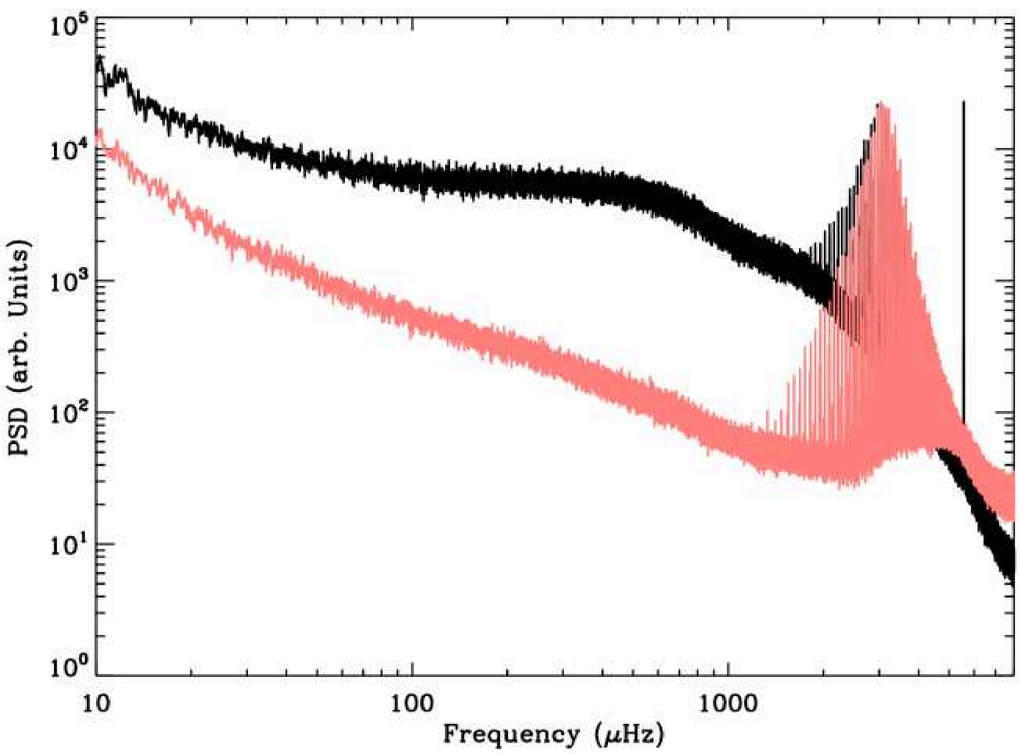
Oscillations with $\omega > L_\ell$ and $\omega > N$ have pressure as the restoring force and are p-modes; on the other hand, if $\omega < L_\ell$ and $\omega < N$, buoyancy is the restorer, and it is a g-mode. In stars like the Sun, the Lamb frequency decreases monotonically towards the surface, while N has a sharp peak near the center, and then rapidly drops to zero.

• The solar oscillations, by definition, very slightly distort the shape of the Sun.



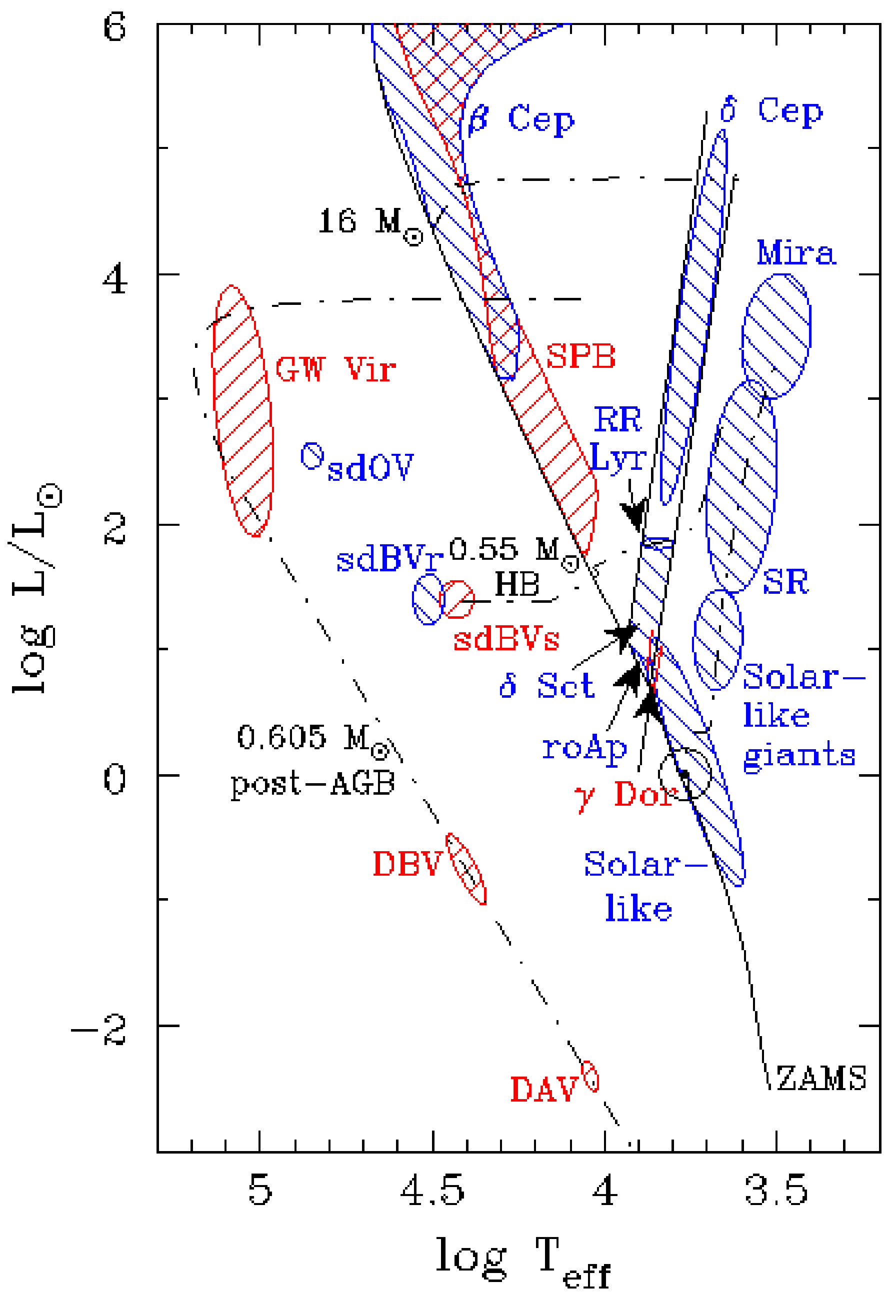
Asteroseismology

The pressure variations connected with the sound waves lead to fluctuations in temperature. This can change both the strength of the spectral lines being observed, and (very slightly) the total broadband intensity. Essentially, because of its p-mode oscillations, the Sun is a variable star which pulsates with an amplitude of ~ 0.01 millimag.



Comparison of solar power spectrum from spectroscopy (red) and photometry (black).

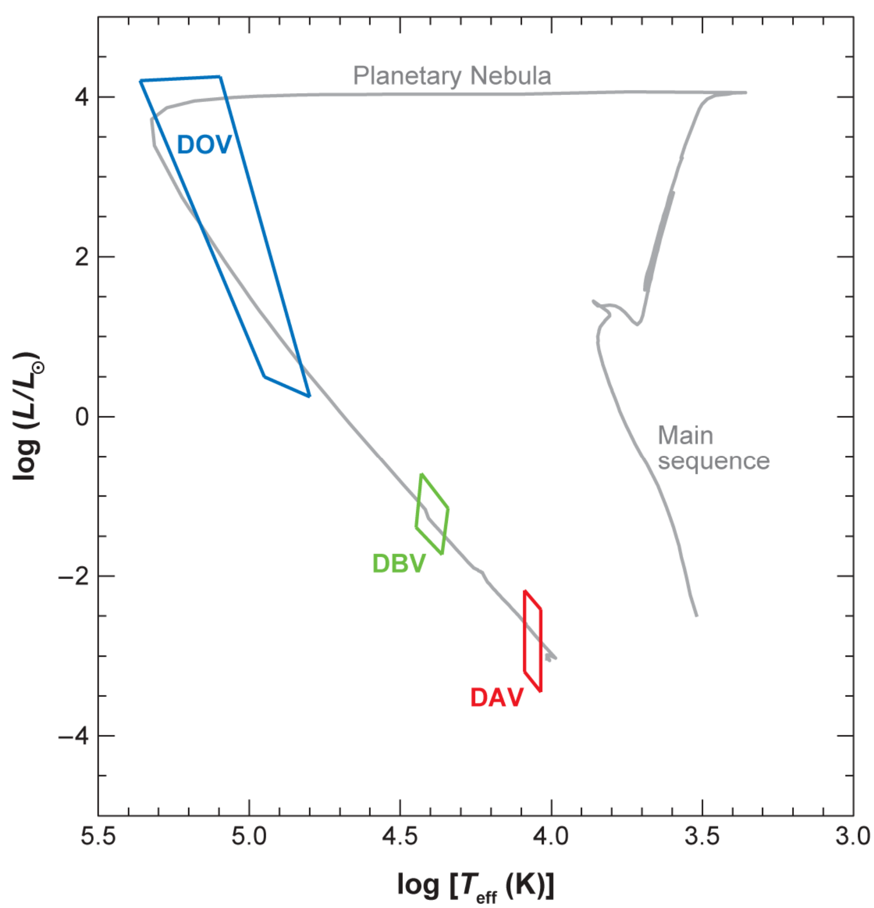
The changes in intensity open up the possibility of performing asteroseismology on stars. There are several types of stars for which such an analysis is viable; the most common of which are white dwarfs, pre-white dwarfs, solar-type stars, and subgiant stars.



White Dwarfs and Pre-White Dwarfs

Aside from the Sun, the first non-radial pulsators to be analyzed by asteroseismological techniques were white dwarfs. Since white dwarfs (and planetary nebula nuclei) are hot, they do not have convective envelopes. This means that a) there is no driver for p-modes, and b) g-modes can propagate to the surface. As a result, white dwarfs (and pre-white dwarfs) that are in an instability strip (DAs for the partial ionization of helium, DBs for partial ionization of doubly-ionized helium and DOs and planetary nebulae nuclei for the partial ionization of carbon and oxygen) can be non-radial pulsators.

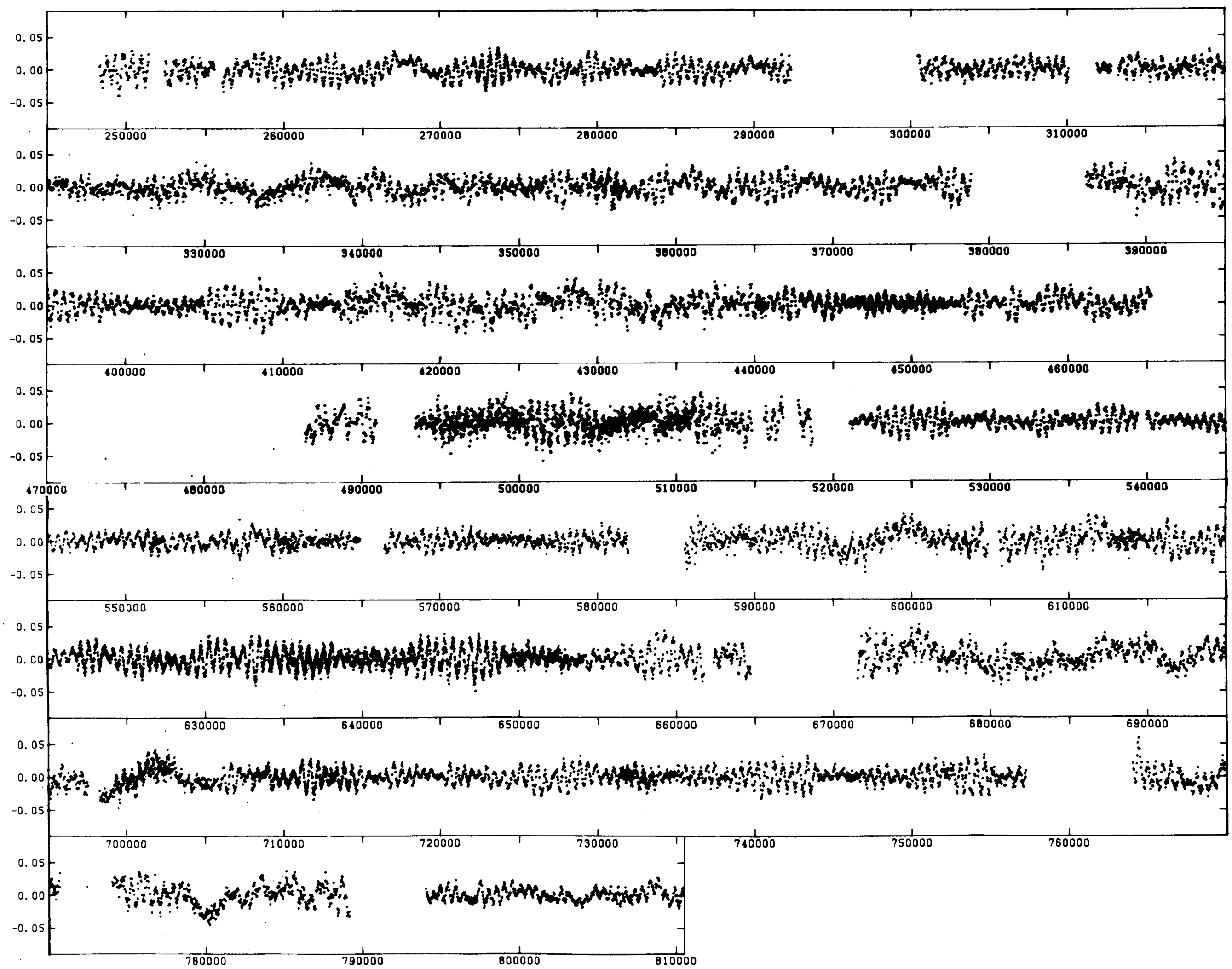
DA white dwarf variables (DAVs) are called ZZ Ceti stars; DOVs are generally called GW Vir or PG 1159 stars. DB variables are usually just called DBVs.



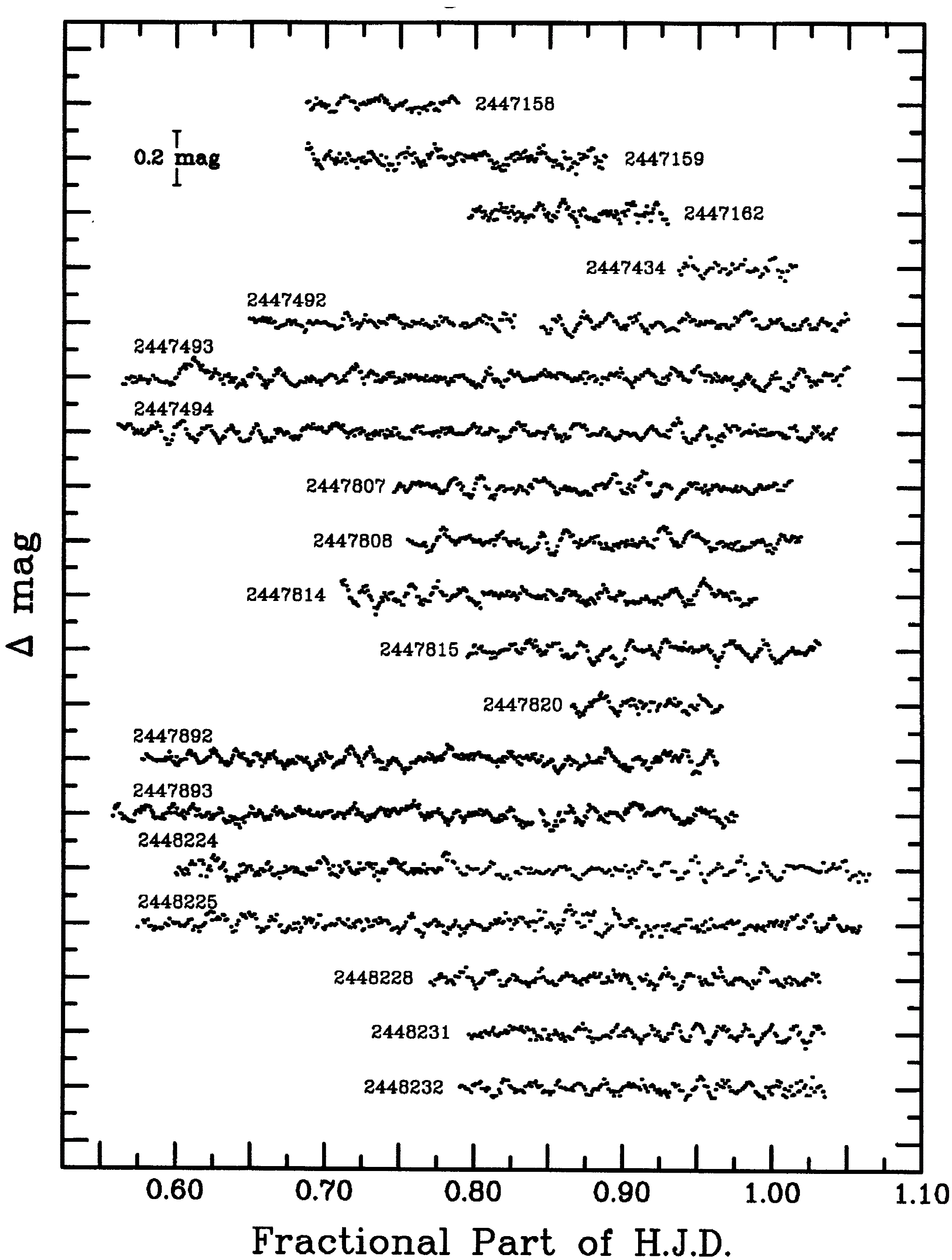
White dwarfs can pulsate with amplitudes of ~ 0.05 mag, and much of their power coming out in periods of ~ 10 min. (For pre-white dwarf planetary nebula central stars, this period is about ~ 25 min.) Moreover, often times, these objects have two principle modes that are moderately close together. That means that the periods beat: every few hours the positive and negative excursions add up, creating variations of almost ~ 0.2 mag. So detecting multi-mode pulsator from the ground is easy! However, in order to extract information about the stars, one needs to resolve all the modes.

A rule of thumb: to resolve two frequencies separated by $\Delta\nu$, one needs to observe (uninterrupted) for $1/\nu$ seconds. So to identify two modes separated by 2 μHz, one needs a month or so of continuous observations!

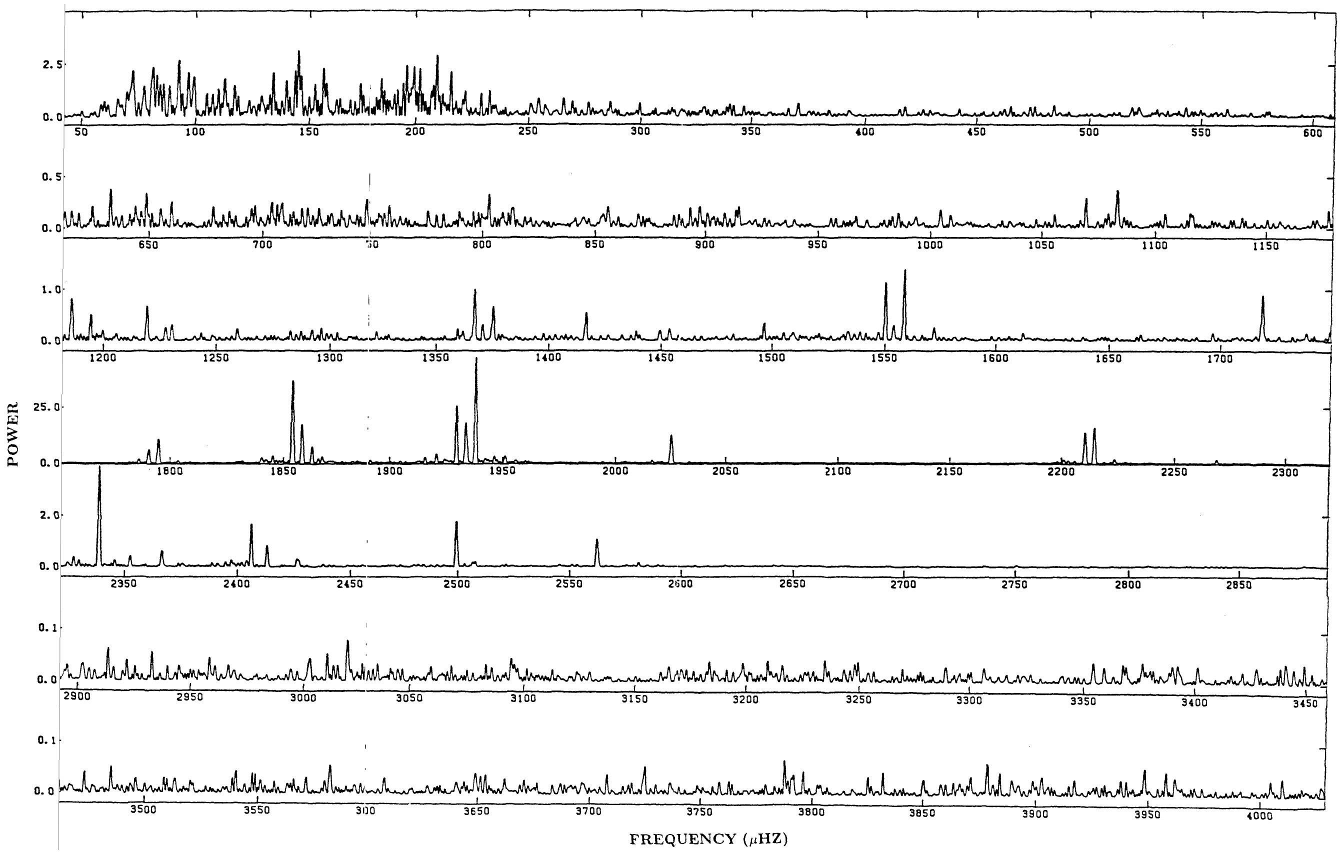
In 1990, the Whole Earth Telescope was formed from a network of small telescopes around the globe. Since then, other networks of telescopes have investigated a small handful of objects.



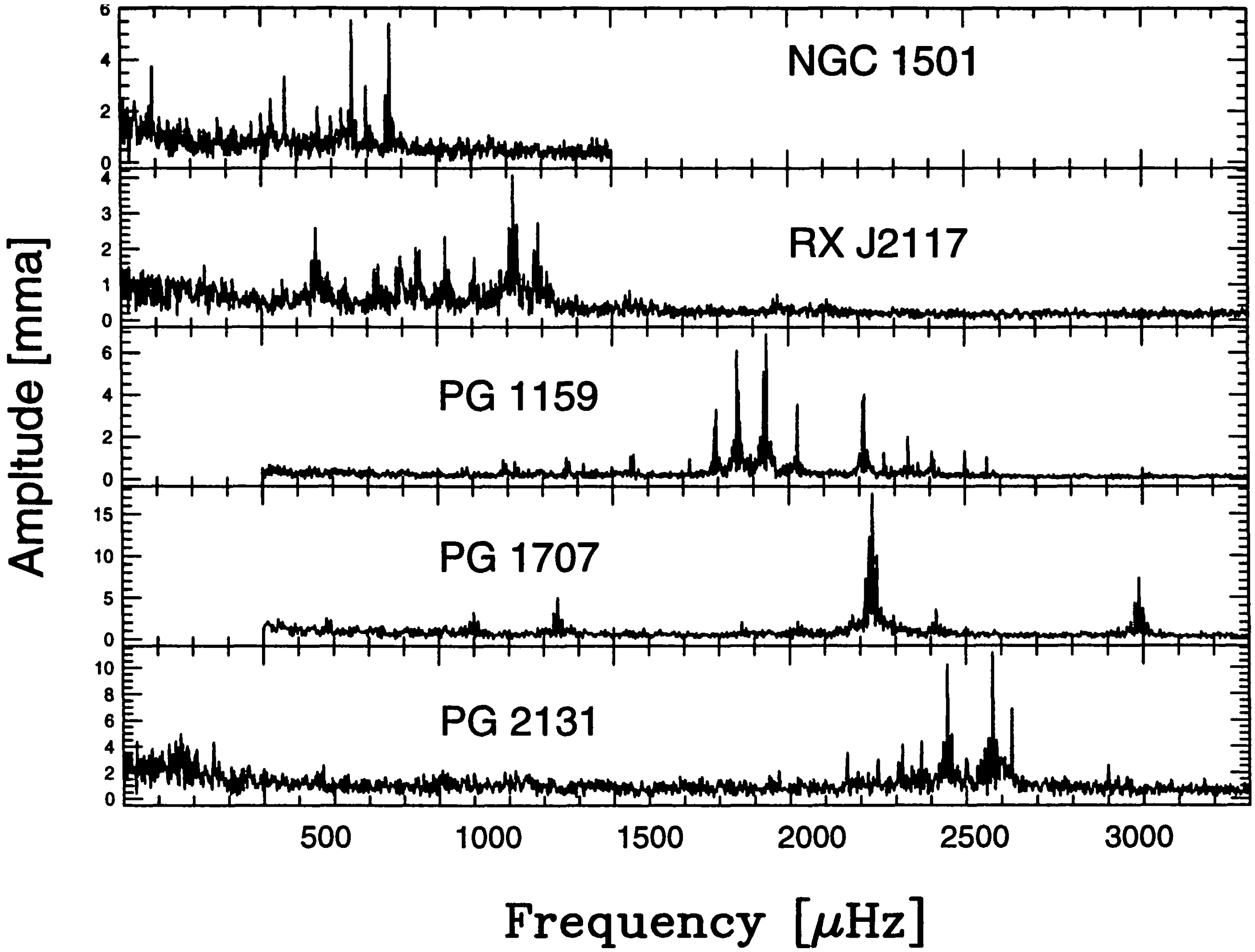
The light curve of white dwarf PG 1159-035 over the central ~ 6 days of a month-long campaign with the Whole Earth Telescope.



2447158
0.2 mag
2447159
2447162
2447434
2447492
2447493
2447494
2447807
2447808
2447814
2447815
2447820
2447892
2447893
2448224
2448225
2448228
2448231
2448232
Δ mag
0.60
0.70
0.80
0.90
1.00
1.10
Fractional Part of H.J.D.



The power spectrum of PG 1159-035.



Recall that g-modes propagate at the Brunt-Väisälä, or buoyancy, frequency which, in the absense of a chemical gradient (see 32.07) is

$$N^2 = -\frac{g\delta}{\lambda_P}\left[\nabla_{\rm ad} - \nabla_{\rm rad}\right] \tag{32.16}$$

Unlike p-modes, which exhibit a periodicity in frequency, g-mode pulsations are more-or-less periodic in their periods. In the limit of high-n, the asymptotic form of the period relation is

$$\Pi_{n,\ell} = \frac{\Pi_0}{\sqrt{\ell(\ell+1)}}(n+\epsilon) \tag{32.16}$$

So the period spacing for $\ell = 1$ modes will be a factor of $\sqrt{3}$ larger than for $\ell = 2$. Π_0 is related to the buoyancy frequency, N, by

$$\Pi_0 = 2\pi^2 \left(\int_{r_1}^{r_2} N \frac{dr}{r} \right)^{-1} \tag{32.17}$$

where r_1 and r_2 are the boundary regions for the mode. For white dwarfs, this quantity depends primary on the total stellar mass, with

$$\Pi_0 = 15.5 \left(\frac{\mathcal{M}}{\mathcal{M}_\odot} \right)^{-1.3} \left(\frac{\mathcal{L}}{100\mathcal{L}_\odot} \right)^{0.035} \tag{32.18}$$

The mass of a well-observed white dwarf can therefore be measured to 3 significant digits. Other information obtainable include the rotation rate, the thickness of the non-degenerate surface layers, and the composition of both the main body of the star and the surface layer.

Finally, there is also the possibility of observing actual stellar evolution via temporal changes in the periods of white dwarfs. As described above, g-modes have a period periodicity. If k is the wave number, then the buoyancy is related to period by

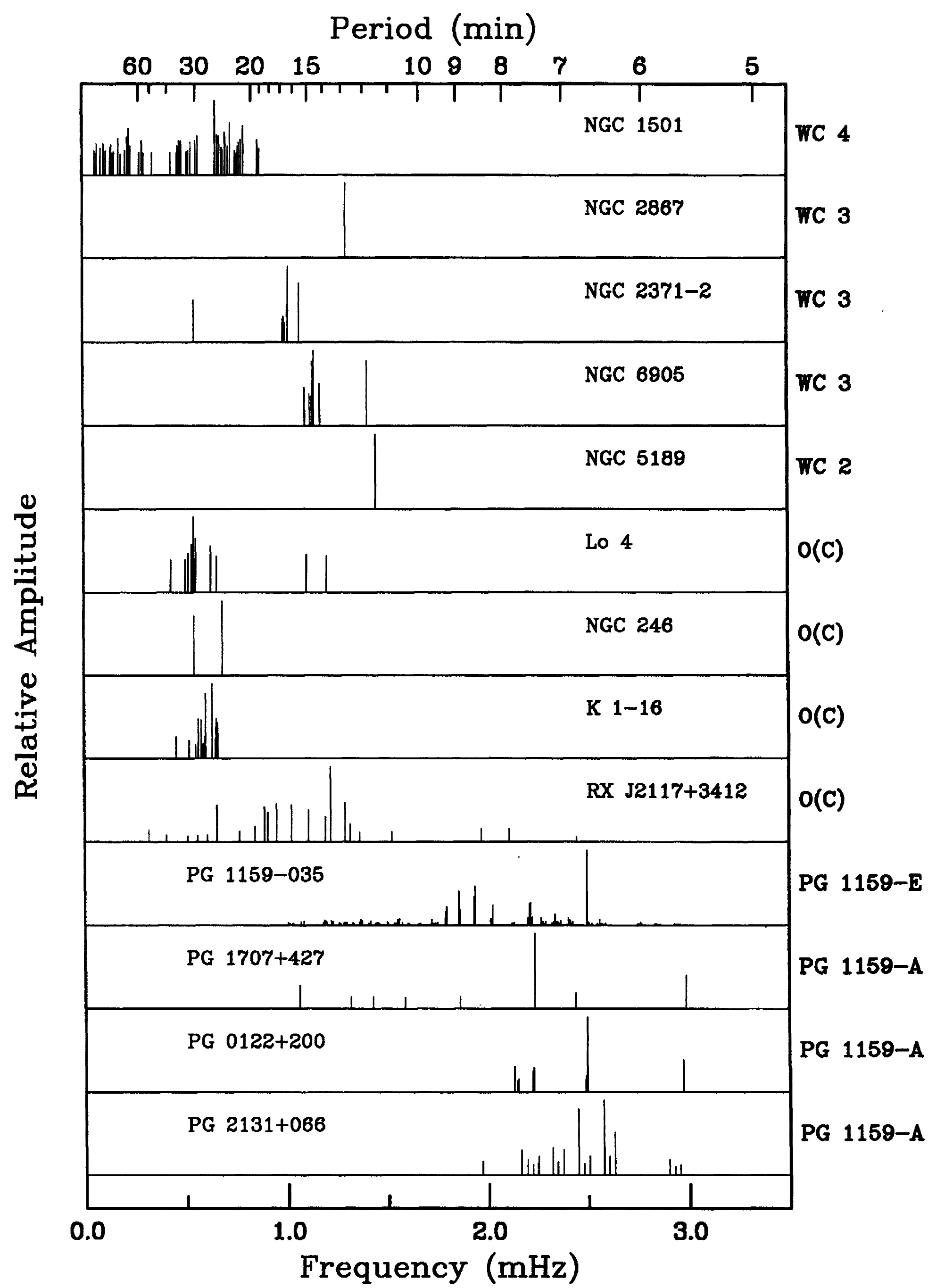
$$\Pi = -\frac{kr}{N} \tag{32.19}$$

For roughly isothermal cores (as might be expected from a white dwarf with Urca process neutrino cooling), one can write down the approximate dependence of period on core temperature and stellar radius

$$\frac{d\ln\Pi}{dt} \sim -\frac{1}{2}\frac{d\ln T_{\rm core}}{dt} + \frac{d\ln R}{dt} \tag{32.20}$$

At first glance, this relation looks problematic, since, as a hot white dwarf evolves, it should both cool and contract, and those two effects work in opposite directions. Fortunately, the contraction of

white dwarfs (and pre-white dwarfs) should be minor, implying that the g-mode periods should decrease with time. This makes empirical sense, as pulsating planetary nebulae have principle periods of $\sim$ 25 min, while white dwarf periods are 5 to 10 min. However, the attempts to directly observe white dwarf cooling have thus far produced ambiguous results.



Solar type Stars

Over the years, there have been many attempts to detect the p-mode oscillations of nearby solar-type stars through ground-based observations. But (with one or two exceptions), it was not until Kepler that such observations became feasible. Now, with the Kepler and CoRoT satellites, we can analyze the p-modes of large numbers of stars and derive their stellar parameters from their general oscillation properties. This is called "Ensemble Asteroseismology."

The power spectrum of most solar-type stars looks similar to that of the Sun. There is a regularity in the frequency separation of power related to the sound travel time across the stellar diameter

$$\Delta\nu = 2\left(\int_0^R \frac{dr}{c_s}\right)^{-1} \propto \langle\rho\rangle^{1/2} \tag{32.10}$$

This is generally called the "large separation". Since $c_s \propto \langle\rho\rangle^{1/2}$, this periodicity generally scales with the square root of the mean density of star: as the star gets larger, $\Delta\nu$ decreases.

There is also another periodicity known as the "small separation" in the power spectrum peaks

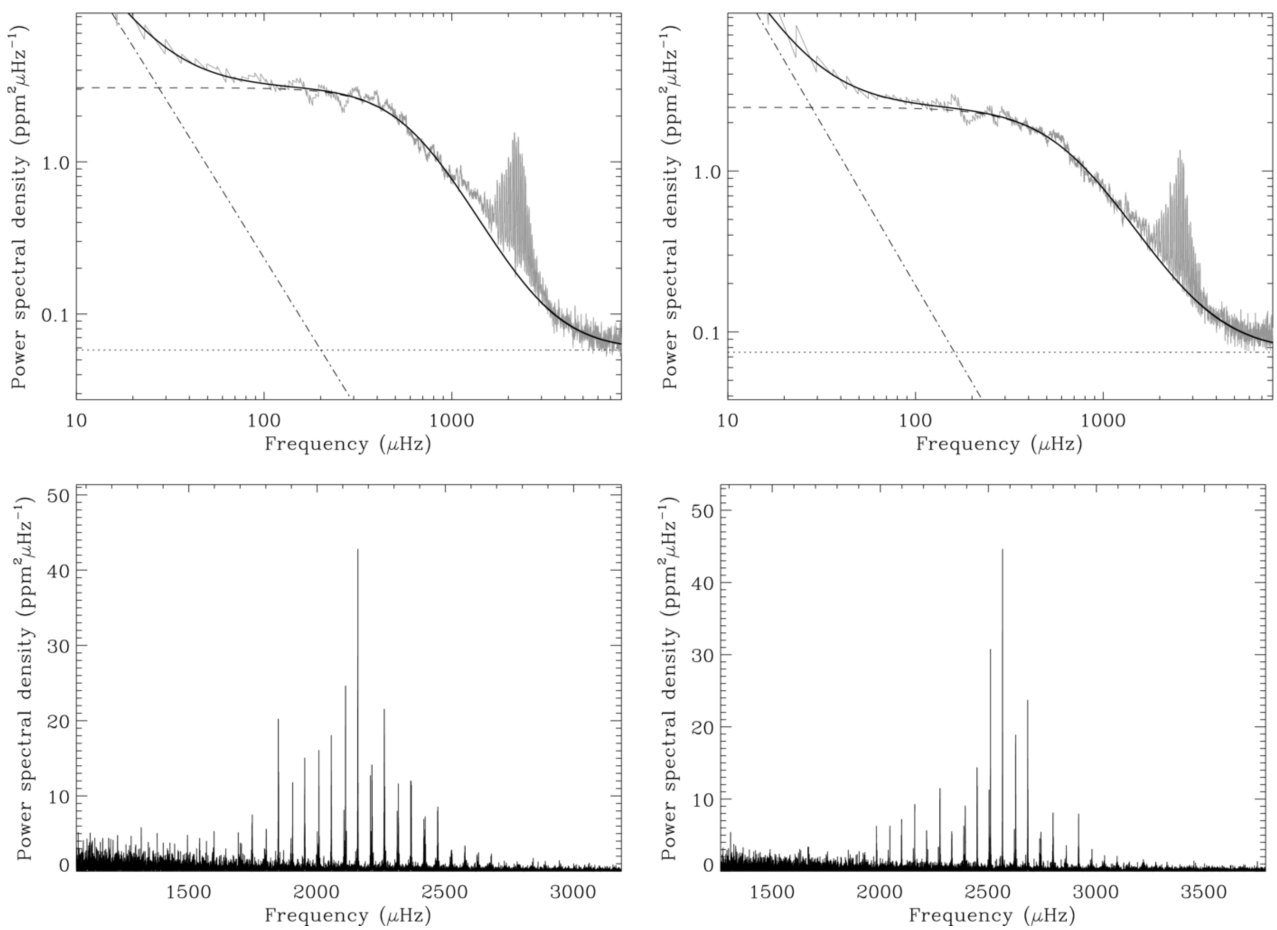
$$\delta\nu_\ell = \nu_{n,\ell} - \nu_{n-1,\ell+2} = (4\ell + 6)D_0 \tag{32.21}$$

where

$$D_0 = -\frac{\Delta\nu}{4\pi^2\nu_{n,\ell}}\left[\frac{c_s(R)}{R} - \int_0^R \frac{dc_s}{dr}\frac{dr}{r}\right] \tag{32.22}$$

This small frequency separation is most sensitive to the sound-speed gradient at small r (i.e., the core), which in turn is sensitive to the core's chemical composition gradient and hence to the star's evolutionary state.

Finally, the overall pattern of power in the oscillations looks approximately Gaussian, with the bulk of the power coming out at a frequency, ν_{max}. This frequency is likely related to the maximum frequency cutoff (32.04). If so, then as a star evolves towards the subgiant branch and its surface gravity decreases, ν_{max} should move towards lower frequencies.



Power spectra for a wide binary system, 16 Cyg A (left) and 16 Cyg B (right). Note the Gaussian-like peak at high-frequency, and the periods in frequency space.

If the peak in the distribution of frequency modes is related to the cutoff frequency (32.04), and through (32.12) $\Delta\nu \propto \langle\rho\rangle^{1/2}$ then a simply scaling to the Sun yields

$$\nu_{\rm max} \propto \nu_{\rm cut} \propto \frac{\mathcal{M}}{R^2 T_{\rm eff}^{1/2}} \Longrightarrow \frac{\nu_{\rm max}}{\nu_{\rm max,\odot}} = \frac{\mathcal{M}/\mathcal{M}_\odot}{(R/R_\odot)^2 \left(T_{\rm eff}/T_{\rm eff,\odot}\right)^{1/2}} \tag{32.23}$$

and

$$\frac{\Delta\nu}{\Delta\nu_\odot} = \sqrt{\frac{\mathcal{M}/\mathcal{M}_\odot}{(R/R_\odot)^3}} \tag{32.24}$$

Putting these two equations together yields

$$\frac{R}{R_\odot} = \left(\frac{\nu_{\rm max}}{\nu_{\rm max,\odot}}\right)\left(\frac{\Delta\nu}{\Delta\nu_\odot}\right)^{-2}\left(\frac{T_{\rm eff}}{T_{\rm eff,\odot}}\right)^{1/2} \tag{32.25}$$

$$\frac{\mathcal{M}}{\mathcal{M}_\odot} = \left(\frac{\nu_{\rm max}}{\nu_{\rm max,\odot}}\right)^3\left(\frac{\Delta\nu}{\Delta\nu_\odot}\right)^{-4}\left(\frac{T_{\rm eff}}{T_{\rm eff,\odot}}\right)^{3/2} \tag{32.26}$$

For the Sun, $\nu_{\rm max} \sim 3090$ μHz, $\Delta\nu \sim 135.1$ μHz, and $T_{\rm eff} \sim 5777$ K. So, without any complex modeling, measurements of $\nu_{\rm max}$ and $\Delta\nu$ can produce estimates of $\mathcal{M}$ and R than are better than $\sim 10\%$.

We can also use the large and small separations in the power spectrum peaks to estimate the age of a solar-type star. Such a measurement requires a set of evolutionary models (to relate quantities such as the mean density and core mean molecular weight to age), but are fairly straightforward. Below is one such set of models for

solar-metallicity stars.

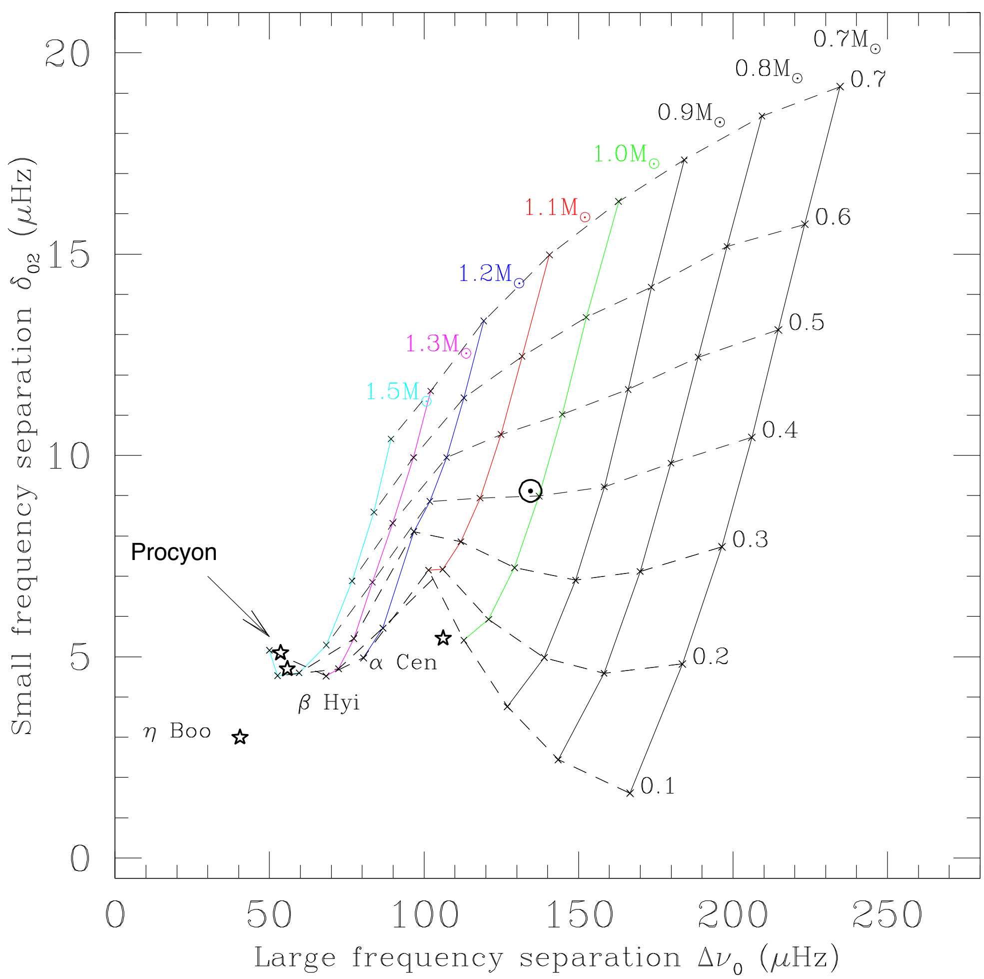


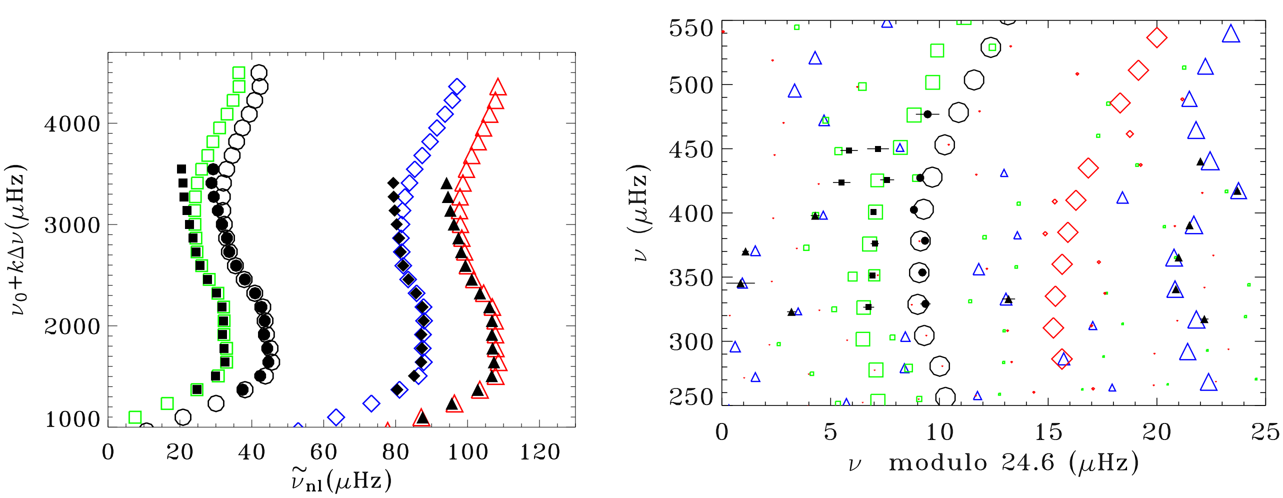
Diagram showing large separation versus small separation for a series of evolutionary models calculated for different masses, but fixed metallicity. Solid lines are evolutionary tracks for decreasing core's hydrogen abundance. Dashed black lines indicate models with same core's hydrogen content decreasing from $X_c = 0.7$ (ZAMS) to $X_c = 0.1$. The Sun and other well-observed stars are marked.

Note: one complication in interpreting the power spectrum of subgiants and red giants is "mixed mode" oscillations. In solar-type objects, interior g-modes have low frequency, while most p-modes have a relatively high frequency, so the two modes do not interact. However, as stars evolve, the large chemical gradients that are built up and the large density gradient between the core and the envelope can increase the buoyancy frequency. These interior g-modes can then interact with p-modes of roughly the same frequency, creating modes of mixed character. As a result, frequencies become shifted from the regular $\Delta\nu$ spacing, and the star's power spectrum has a quite complicated appearance (at least in the region of g-mode and p-mode frequency overlap).

Most p-mode-based estimates of stellar mass, radius, and age are based on ensemble asteroseismology. In fact, ensemble asteroseismology in open clusters are particularly useful, as the technique naturally works best when the stars being considered are similar. For example, by determining the masses of red giants as a function of luminosity in a single cluster, one can measure mass-loss directly, and not be affected by systematic uncertainties due to stellar metallicity, and the like. (In theory, one could have also done this with globular clusters, if there were a suitable such object in a Kepler field. But crowding and distance more-or-less excluded this possibility).

An alternative and more precise way of determining stellar parameters is through the analysis of individual oscillation frequecies, rather than the globular parameters of $\nu_{\max}, \Delta\nu$, and $\delta\nu_\ell$. This requires the computation of detailed stellar models for each star, in order to compare theoretical oscillation frequencies with the observed frequencies. Obviously, this involves a lot more work, but it typically doubles the precision of a mass, radius, and age estimate.

The comparison of observed frequencies versus theoretically computed frequencies is often performed using an "échelle" diagram. For the diagram, a power spectrum is divided into segments of length $\Delta\nu$ and each segment is plotted right above the other. If the $\Delta\nu$ spacing were perfectly regular, the result would be a verticle line in the diagram. Curvature or other departures from this line contain the detailed information about the model. (In particular, points which are well off the ridge lines represent mixed-mode frequencies. Since mode-mixing increases as the star evolves, the presence of such oscillations provides information about the evolutionary state of the star. In fact, by matching the observed and theoretical frequencies in an échelle diagram, it is possible to fix the mass and radius of a star to $\sim 2\%$ and the age to $\sim 7\%$.



An échelle diagram for the Sun (left) and red giant star KIC4351319 (right). The filled points are observations, while the open shapes are theoretical calculations. Points will large deviations from the ridge lines represente mixed-mode oscillations. Circles represent modes with $\ell = 0$, triangles show $\ell = 1$, squares $\ell = 2$, and diamonds $\ell = 3$. The distance between two adjacent frequency columns represents the small separation, $\delta\nu_\ell$.

Precision of Parameter Estimates

	Radius	Mass	Age
Sun-Like Main Sequence			
CMD Diagram	...	...	$\sim 15\%$
Ensemble Asteroseismology	$\sim 6\%$	$\sim 6\%$	...
Individual Frequencies	0.3%	0.8%	3.7%
Evolved Stars			
CMD Diagram	...	...	...
Ensemble Asteroseismology	$\sim 15\%$	$\sim 15\%$	...
Individual Frequencies	1%	2%	7%

Stellar Rotation

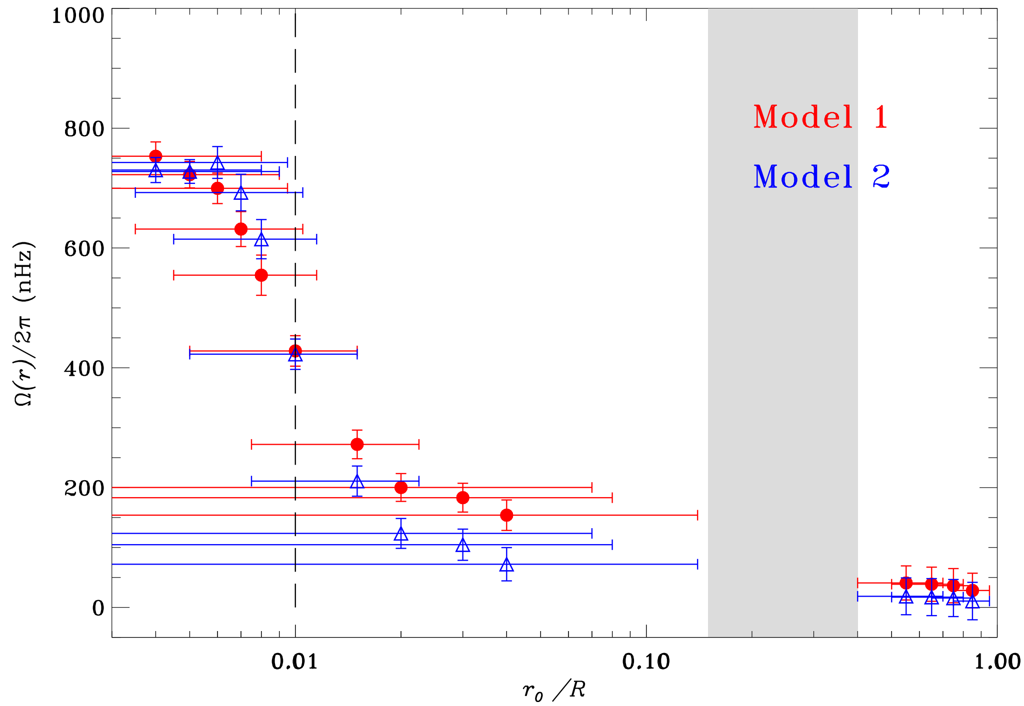
For a static star, the orientation of the star is irrelevant, and all the m modes are degenerate. But, just like the Zeeman and Stark splitting in atoms, the inclusion of a preferred direction (in this case, the rotation axis) causes the m modes to be split into $2\ell+1$ different components, i.e.,

$$\delta\nu_{n,\ell,m} = \nu_{n,\ell,m} - \nu_{n,\ell,0} \tag{32.27}$$

As long as the rotation is slow enough so that the centrifugal force is negligible, (32.13) and (32.14) hold. With a long enough baseline (as provided by Kepler), the splitting of these modes can be measured, and, by comparison to predictions of stellar models, a star's rotation, as a function of radius, can be determined. This is best done with sub-giants and red giants because

1) Mode-mixing allows us to constrain the star's interior g-modes.

2) The evolution off the main sequence (and the development of shell burning) decouples the core from the surface, allowing both regions to evolve independently. Due to core contraction and envelope expansion, the core rotation should speed up, the surface rotation should slow down, with the hydrogen-burning shell dividing the two zones. This is confirmed by observations, though the theoretical models predict rotation rates that are an order of magnitude faster than what is inferred from seismology.



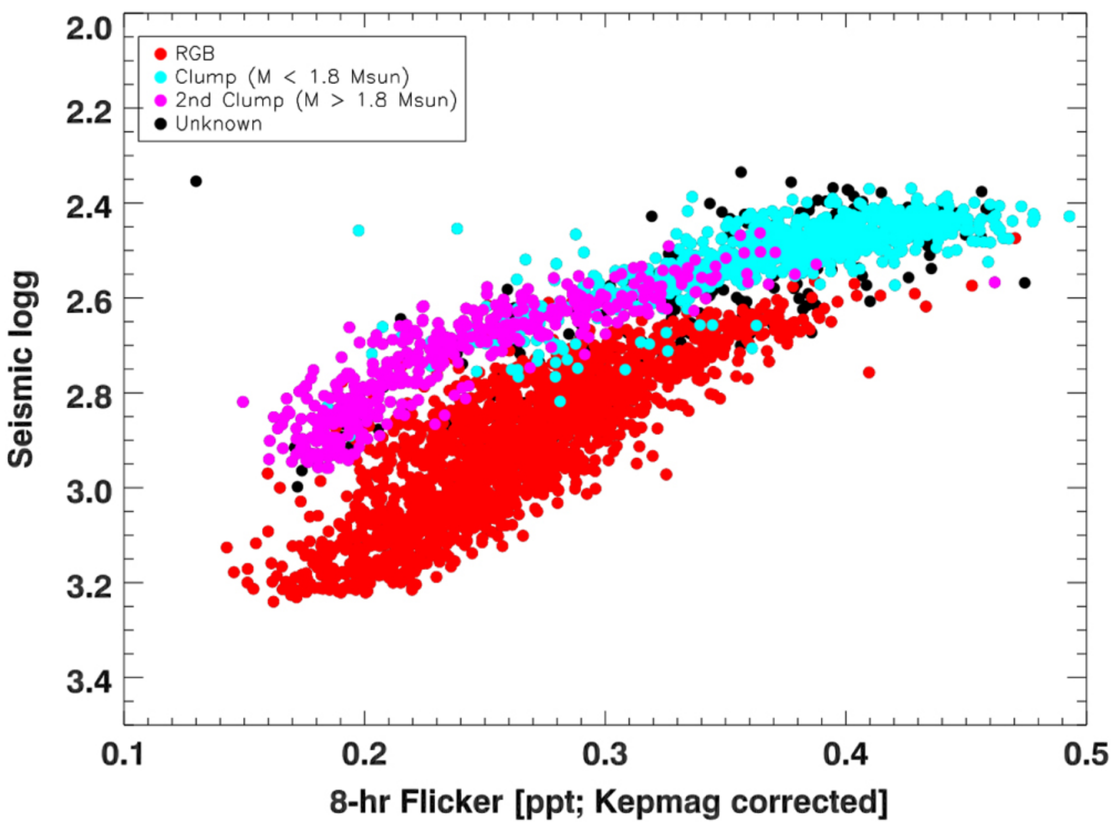
Two models for the internal rotation of RGB star KIC 4448777 based on asteroseismology. The dashed line shows the hydrogen-burning shell, while the shaded region indicators regions where no solutions could be found. In general, the core of RGB stars rotates 8 to 17 faster than the outer layers, and the mean rotation slows as the star ascends the giant branch.

Pulsation Amplitudes

There is still no theory to predict the amplitude of stellar pulsations. (While there are arguments for how the excitation of pulsations scales with stellar conditions, our understanding of damping is poor.) Using empirical Kepler data, one can express the oscillation amplitude as a function of luminosity and mass, i.e.,

$$A \propto \frac{L^s}{\mathcal{M}^t}$$

with $s \sim 0.9$ and $t \sim 1.9$. Alternatively, the pulsation amplitude can be expressed using a "flicker" parameter which scales with surface gravity.



Either way, it appears that as stars evolve off the main sequence towards the giant branch, their oscillation amplitudes increase.